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# The QCD Pomeron with Optimal Renormalization[1]

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## Abstract

It is shown that the next-to-leading order (NLO) corrections to the QCD Pomeron intercept obtained from the BFKL equation, when evaluated in non-Abelian physical renormalization schemes with BLM optimal scale setting do not exhibit the serious problems encountered in the $\overline{\mathrm{MS}}$-scheme. A striking feature of the NLO BFKL Pomeron intercept in the BLM approach is its rather weak dependence on the virtuality of the reggeized gluon. This remarkable property yields an important approximate conformal invariance. The results obtained provide an opportunity for applications of NLO BFKL resummation to high-energy phenomenology.

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The discovery of rapidly increasing structure functions in deep inelastic scattering (DIS) at HERA [1] at small-x is in agreement with the expectations of the QCD high-energy limit. The Balitsky-Fadin-Kuraev-Lipatov (BFKL) [2] resummation of energy logarithms is anticipated to be an important tool for exploring this limit. The leading order (LO) BFKL calculations [2] predict a steep rise of QCD cross sections. Namely, the highest eigenvalue, $\omega^{max}$, of the BFKL equation [2] is related to the intercept of the Pomeron which in turn governs the high-energy asymptotics of the cross sections: $\sigma \sim s^{\alpha_{I\!P}-1} = s^{\omega^{max}}$. The BFKL Pomeron intercept in the LO turns out to be rather large: $\alpha_{I\!P} - 1 = \omega_L^{max} = 12 \ln 2 \, (\alpha_S/\pi) \simeq 0.55$ for $\alpha_S = 0.2$; hence, it is very important to know the next-to-leading order (NLO) corrections. In addition, the LO BFKL calculations have restricted phenomenological applications because, *e.g.*, the running of the QCD coupling constant $\alpha_S$ is not included, and the kinematic range of validity of LO BFKL is not known.

Recently the NLO corrections to the BFKL resummation of energy logarithms were calculated; see Refs. [3, 4] and references therein. The NLO corrections [3, 4] to the highest eigenvalue of the BFKL equation turn out to be negative and even larger than the LO contribution for $\alpha_S > 0.157$. In such circumstances the phenomenological significance of the NLO BFKL calculations seems to be rather obscure.

However, one should stress that the NLO calculations, as any finite-order perturbative results, contain both renormalization scheme and renormalization scale ambiguities. The NLO BFKL calculations [3, 4] were performed by employing the modified minimal subtraction scheme ($\overline{\rm MS}$) [5] to regulate the ultraviolet divergences with arbitrary scale setting.

In this work we consider the NLO BFKL resummation of energy logarithms [3, 4] in physical renormalization schemes in order to study the renormalization scheme dependence. To resolve the renormalization scale ambiguity we utilize Brodsky-Lepage-Mackenzie (BLM) optimal scale setting [6]. We show that the reliability of QCD predictions for the intercept of the BFKL Pomeron at NLO when evaluated using BLM scale setting within non-Abelian physical schemes, such as the momentum space subtraction (MOM) scheme [7, 8] or the $\Upsilon$-scheme based on $\Upsilon \to ggg$ decay, is significantly improved compared to the $\overline{\rm MS}$-scheme. This provides a basis for applications of NLO BFKL resummation to high-energy phenomenology.

We begin with the representation of the $\overline{\rm MS}$-result of NLO BFKL [3, 4] in physical renormalization schemes. Although the $\overline{\rm MS}$-scheme is somewhat artificial and lacks a clear physical picture, it can serve as a convenient intermediate renormalization scheme. The eigenvalue of the NLO BFKL equation at transferred momentum squared $t = 0$ in the $\overline{\rm MS}$-scheme [3, 4] can be represented as the action of the NLO BFKL kernel (averaged over azimuthal angle) on the LO eigenfunctions $(Q_2^2/Q_1^2)^{-1/2+i\nu}$ [3]:

$$\begin{aligned}
\omega_{\overline{MS}}(Q_1^2, \nu) &= \int d^2 Q_2 \;\; K_{\overline{MS}}(\vec{Q}_1, \vec{Q}_2) \left( \frac{Q_2^2}{Q_1^2} \right)^{-\frac{1}{2}+i\nu} = \\
&= N_C \chi_L(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \left[ 1 + r_{\overline{MS}}(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \right], \qquad (1)
\end{aligned}$$

where

$$\chi_L(\nu) = 2\psi(1) - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)$$

is the function related with the LO eigenvalue, $\psi = \Gamma'/\Gamma$ denotes the Euler $\psi$-function, the $\nu$-variable is conformal weight parameter [9], $N_C$ is the number of colors, and $Q_{1,2}$ are the virtualities of the reggeized gluons.

The calculations of Refs. [3, 4] allow us to decompose the NLO coefficient $r_{\overline{MS}}$ of Eq. (1) into $\beta$-dependent and the conformal ($\beta$-independent) parts:

$$r_{\overline{MS}}(\nu) = r^{\beta}_{\overline{MS}}(\nu) + r^{conf}_{\overline{MS}}(\nu), \tag{2}$$

where

$$r^{\beta}_{\overline{MS}}(\nu) = -\frac{\beta_0}{4}\left[\frac{1}{2}\chi_L(\nu) - \frac{5}{3}\right] \tag{3}$$

and

$$\begin{aligned} r^{conf}_{\overline{MS}}(\nu) \;\; = \;\; & -\frac{N_C}{4\chi_L(\nu)}\left[\frac{\pi^2\sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3 + \left(1 + \frac{N_F}{N_C^3}\right)\frac{11 + 12\nu^2}{16(1+\nu^2)}\right) - \chi''_L(\nu)\right. \\ & \left. + \frac{\pi^2 - 4}{3}\chi_L(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} - 6\zeta(3) + 4\varphi(\nu)\right] \end{aligned} \tag{4}$$

with

$$\varphi(\nu) = 2\int_0^1 dx \frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6} - \mathrm{Li}_2(x)\right], \; \mathrm{Li}_2(x) = -\int_0^x dt \frac{\ln(1-t)}{t}. \tag{5}$$

Here $\beta_0 = (11/3)N_C - (2/3)N_F$ is the leading coefficient of the QCD $\beta$-function, $N_F$ is the number of flavors, $\zeta(n)$ stands for the Riemann zeta-function, $\mathrm{Li}_2(x)$ is the Euler dilogarithm (Spence-function). In Eq. (4) $N_F$ denotes flavor number of the Abelian part of the $gg \to q\overline{q}$ process contribution. The Abelian part is not associated with the running of the coupling [10] and is consistent with the correspondent QED result for the $\gamma^*\gamma^* \to e^+e^-$ cross section [11].

The $\beta$-dependent NLO coefficient $r^{\beta}_{\overline{MS}}(\nu)$, which is related to the running of the coupling, receives contributions from the gluon reggeization diagrams, from the virtual part of the one-gluon emission, from the real two-gluon emission, and from the non-Abelian part [10] of the $gg \to q\overline{q}$ process. There is an omitted term in $r^{\beta}_{\overline{MS}}(\nu)$ proportional to $\chi'_L(\nu)$ which originates from the asymmetric treatment of $Q_1$ and $Q_2$ and which can be removed by the redefinition of the LO eigenfunctions [3].

The NLO BFKL Pomeron intercept then reads for $N_C = 3$: [3]

$$\alpha^{\overline{MS}}_{I\!P} - 1 = \omega_{\overline{MS}}(Q^2, 0) = 12\,\ln 2\,\frac{\alpha_{\overline{MS}}(Q^2)}{\pi}\left[1 + r_{\overline{MS}}(0)\frac{\alpha_{\overline{MS}}(Q^2)}{\pi}\right], \tag{6}$$

$$r_{\overline{MS}}(0) \simeq -20.12 - 0.1020 N_F + 0.06692\beta_0, \tag{7}$$

$$r_{\overline{MS}}(0)_{|N_F=4} \simeq -19.99.$$

Physical renormalization schemes provide small and physically meaningful perturbative coefficients by incorporating large corrections into the definition of the coupling constant. One of the most popular physical schemes is MOM-scheme [7, 8], based on renormalization of the triple-gluon vertex at some symmetric off-shell momentum. However, in the MOM-scheme the coupling constant is gauge-dependent already in the LO, and there are rather cumbersome technical difficulties. These difficulties can be avoided by performing calculations in the intermediate $\overline{\mathrm{MS}}$-scheme, and then by making the transition to some physical scheme by a finite renormalization [7]. In order to eliminate the dependence on gauge choice and other theoretical conventions, one can consider renormalization schemes based on physical processes [6], *e.g.*, V-scheme based on heavy quark potential. Alternatively, one can introduce a physical scheme based on $\Upsilon \to ggg$ decay using the NLO calculations of Ref. [12].

A finite renormalization due to the change of scheme can be accomplished by a transformation of the QCD coupling [7]:

$$\alpha_S \to \alpha_S \Big[ 1 + T \frac{\alpha_S}{\pi} \Big], \tag{8}$$

where T is some function of $N_C$, $N_F$, and for the MOM-scheme, of a gauge parameter $\xi$. Then the NLO BFKL eigenvalue in the MOM-scheme can be represented as follows

$$\begin{aligned} \omega_{MOM}(Q^2,\nu) &= N_C \chi_L(\nu) \frac{\alpha_{MOM}(Q^2)}{\pi} \left[ 1 + r_{MOM}(\nu) \; \frac{\alpha_{MOM}(Q^2)}{\pi} \right], \\ r_{MOM}(\nu) &= r_{\overline{MS}}(\nu) + T_{MOM}. \end{aligned} \tag{9}$$

For the transition from the $\overline{\mathrm{MS}}$-scheme to the MOM-scheme the corresponding T-function has the following form [7]:

$$\begin{aligned} T_{MOM} &= T_{MOM}^{conf} + T_{MOM}^{\beta}, \\ T_{MOM}^{conf} &= \frac{N_C}{8} \Big[ \frac{17}{2} I + \xi \frac{3}{2}(I-1) + \xi^2 (1 - \frac{1}{3} I) - \xi^3 \frac{1}{6} \Big], \\ T_{MOM}^{\beta} &= -\frac{\beta_0}{2} \Big[ 1 + \frac{2}{3} I \Big], \end{aligned} \tag{10}$$

where $I = -2 \int_0^1 dx \ln(x)/[x^2 - x + 1] \simeq 2.3439$.

Analogously, one can obtain for the V-scheme [6]:

$$T_V = \frac{2}{3} N_C - \frac{5}{12} \beta_0, \tag{11}$$

and by the use of the results of Ref. [12] for the $\Upsilon$-scheme:

$$T_\Upsilon = \frac{6.47}{3} N_C - \frac{2.77}{3} \beta_0. \tag{12}$$

| Scheme | | $T = T^{conf} + T^{\beta}$ | $r(0) = r^{conf}(0) + r^{\beta}(0)$ | $r(0)$ $(N_F = 4)$ |
|---|---|---|---|---|
| M | $\xi = 0$ | $7.471 - 1.281\beta_0$ | $-12.64 - 0.1020N_F - 1.214\beta_0$ | -22.76 |
| O | $\xi = 1$ | $8.247 - 1.281\beta_0$ | $-11.87 - 0.1020N_F - 1.214\beta_0$ | -21.99 |
| M | $\xi = 3$ | $8.790 - 1.281\beta_0$ | $-11.33 - 0.1020N_F - 1.214\beta_0$ | -21.44 |
| V | | $2 - 0.4167\beta_0$ | $-18.12 - 0.1020N_F - 0.3497\beta_0$ | -21.44 |
| $\Upsilon$ | | $6.47 - 0.923\beta_0$ | $-13.6 - 0.102N_F - 0.856\beta_0$ | -21.7 |

Table 1: Scheme-transition function and the NLO BFKL coefficient in physical schemes.

One can see from Table 1 that the problem of a large NLO BFKL coefficient remains. The large size of the perturbative corrections leads to significant renormalization scale ambiguity.

The renormalization scale ambiguity problem can be resolved if one can optimize the choice of scales and renormalization schemes according to some sensible criteria. In the BLM optimal scale setting [6], the renormalization scales are chosen such that all vacuum polarization effects from the QCD $\beta$-function are resummed into the running couplings. The coefficients of the perturbative series are thus identical to the perturbative coefficients of the corresponding conformally invariant theory with $\beta = 0$. The BLM approach has the important advantage of resumming the large and strongly divergent terms in the perturbative QCD series which grow as $n![\alpha_S \beta_0]^n$, *i.e.*, the infrared renormalons associated with coupling constant renormalization. The renormalization scales in the BLM approach are physical in the sense that they reflect the mean virtuality of the gluon propagators [6].

BLM scale setting [6] can be applied within any appropriate physical scheme. In the present case one can show that within the V-scheme (or the $\overline{\mathrm{MS}}$-scheme) the BLM procedure does not change significantly the value of the NLO coefficient $r(\nu)$. This can be understood since the V-scheme, as well as $\overline{\mathrm{MS}}$-scheme, are adjusted primarily to the case when in the LO there are dominant QED (Abelian) type contributions, whereas in the BFKL case there are important LO gluon-gluon (non-Abelian) interactions.

Therefore, from the point of view of BLM scale setting, one can separate QCD processes into two classes specifying whether gluons are involved to the LO or not. Thus one can expect that in the BFKL case it is appropriate to use a physical scheme which is adjusted for non-Abelian interactions in the LO. One can choose the MOM-scheme based on the symmetric triple-gluon vertex [7, 8] or the $\Upsilon$-scheme based on $\Upsilon \to ggg$ decay. The importance of taking into account this circumstance for vacuum polarization effects one can be seen from the "incorrect" sign of the $\beta_0$-term for $r_{\overline{MS}}$ in the unphysical $\overline{\mathrm{MS}}$-scheme (Eq. (7)).

| Scheme | | $r_{BLM}(0)$ ($N_F=4$) | $\alpha_{I\!P}^{BLM}-1=\omega_{BLM}(Q^2,0)$ | | |
|---|---|---|---|---|---|
| | | | $Q^2=1$ GeV$^2$ | $Q^2=15$ GeV$^2$ | $Q^2=100$ GeV$^2$ |
| M | $\xi=0$ | -13.05 | 0.134 | 0.155 | 0.157 |
| O | $\xi=1$ | -12.28 | 0.152 | 0.167 | 0.166 |
| M | $\xi=3$ | -11.74 | 0.165 | 0.175 | 0.173 |
| $\Upsilon$ | | -14.01 | 0.133 | 0.146 | 0.146 |

Table 2: The NLO BFKL Pomeron intercept in the BLM scale setting within non-Abelian physical schemes.

Adopting BLM scale setting, the NLO BFKL eigenvalue in the MOM-scheme is

$$\omega_{BLM}^{MOM}(Q^2,\nu)=N_C\chi_L(\nu)\frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi}\left[1+r_{BLM}^{MOM}(\nu)\frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi}\right], \tag{13}$$

$$r_{BLM}^{MOM}(\nu)=r_{MOM}^{conf}(\nu)\,. \tag{14}$$

The $\beta$-dependent part of the $r_{MOM}(\nu)$ defines the corresponding BLM optimal scale

$$Q_{BLM}^{MOM\,2}(\nu)=Q^2\exp\left[-\frac{4r_{MOM}^{\beta}(\nu)}{\beta_0}\right]=Q^2\exp\left[\frac{1}{2}\chi_L(\nu)-\frac{5}{3}+2\Big(1+\frac{2}{3}I\Big)\right]. \tag{15}$$

Taking into account the fact that $\chi_L(\nu)\to -2\ln(\nu)$ at $\nu\to\infty$, one obtains at large $\nu$

$$Q_{BLM}^{MOM\,2}(\nu)=Q^2\frac{1}{\nu}\exp\left[2\Big(1+\frac{2}{3}I\Big)-\frac{5}{3}\right]. \tag{16}$$

At $\nu=0$ we have $Q_{BLM}^{MOM\,2}(0)=Q^2(4\exp[2(1+2I/3)-5/3])\simeq Q^2\,127$. Note that $Q_{BLM}^{MOM\,2}(\nu)$ contains a large factor, $\exp[-4T_{MOM}^{\beta}/\beta_0]=\exp[2(1+2I/3)]\simeq 168$, which reflects a large kinematic difference between MOM- and $\overline{\rm MS}$- schemes [13, 6], even in an Abelian theory.

Analogously one can implement the BLM scale setting in the $\Upsilon$-scheme (Table 2).

Figs. 1 and 2 give the results for the eigenvalue of the NLO BFKL kernel. We have used the QCD parameter $\Lambda=0.1$ GeV which corresponds to $\alpha_S=4\pi/[\beta_0\ln(Q^2/\Lambda^2)]\simeq 0.2$ at $Q^2=15$ GeV$^2$. Also, the generalization [14, 15] of the $\beta$ -function in the running coupling and of flavor number for continuous treatment of quark thresholds has been used.

One can see from Fig. 1, that the maximum which occurs at non-zero $\nu$ is not as pronounced in the BLM approach compared to the $\overline{\rm MS}$-scheme, and thus it will not serve as a good saddle point at high energies.

One of the striking features of this analysis is that the NLO value for the intercept of the BFKL Pomeron, improved by the BLM procedure, has a very weak dependence on the gluon virtuality $Q^2$. This agrees with the conventional Regge-theory where one

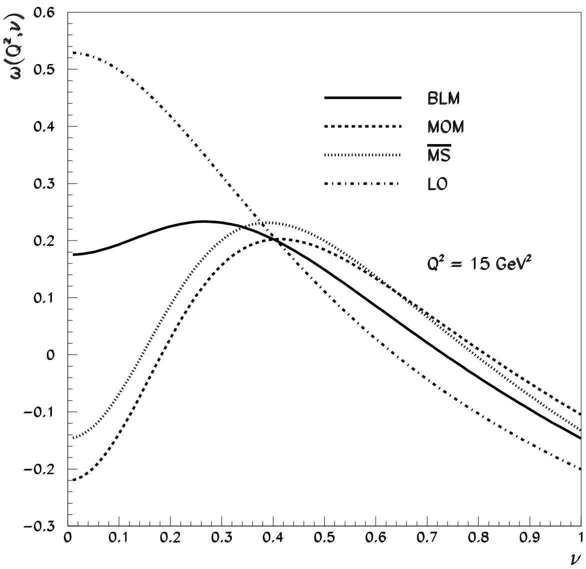


Figure 1: $\nu$-dependence of the NLO BFKL eigenvalue at $Q^2 = 15$ GeV$^2$: BLM (in MOM-scheme) – solid, MOM-scheme (Yennie gauge: $\xi = 3$) – dashed, $\overline{\text{MS}}$-scheme – dotted. LO BFKL ($\alpha_S = 0.2$) – dash-dotted.

expects an universal intercept of the Pomeron without any $Q^2$-dependence. The minor $Q^2$-dependence obtained, on one side, provides near insensitivity of the results to the precise value of $\Lambda$, and, on the other side, leads to approximate scale and conformal invariance. Thus one may use conformal symmetry [9, 16] for the continuation of the present results to the case $t \neq 0$.

Therefore, by the applying of the BLM scale setting within the non-Abelian physical schemes (MOM- and $\Upsilon$- schemes) we do not face the serious problems [17, 18, 19] which were present in the $\overline{\text{MS}}$-scheme, e.g., oscillatory cross section disbehavior based on the saddle point approximation [17], and the somewhat puzzling analytic structure [18] of the $\overline{\text{MS}}$-scheme result [3, 4].

Now we will briefly consider NLO BFKL within other approaches to the optimization of perturbative theory, namely, fast apparent convergence (FAC) [20] and the principle of minimal sensitivity (PMS) [21].

By the use of the FAC [20] one can obtain

$$\omega_{FAC}(Q^2, \nu) = N_C \chi_L(\nu) \frac{\alpha_S(Q^2_{FAC}(\nu))}{\pi}, \tag{17}$$

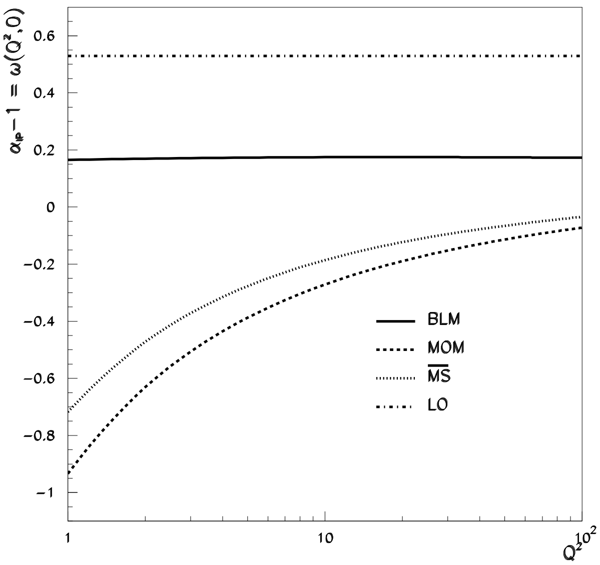


Figure 2: $Q^2$-dependence of the BFKL Pomeron intercept in the NLO. The notation is as in Fig. 1.

$$Q^2_{FAC}(\nu) = Q^2 \exp\left[-\frac{4}{\beta_0} r(\nu)\right]. \tag{18}$$

In the $\overline{\rm MS}$-scheme at $\nu = 0$, $\omega_{FAC} = 0.33 - 0.26$ for $Q^2 = 1 - 100$ GeV$^2$. However, the NLO coefficient $r(\nu)$, and hence, the FAC effective scale, each have a singularity at $\nu_0 \simeq 0.6375$ due to a zero of the $\chi_L(\nu)$-function.

In the PMS approach [21] the NLO BFKL eigenvalue reads as follows

$$\omega_{PMS}(Q^2,\nu) = N_C\, \chi_L(\nu) \frac{\alpha_{PMS}(Q^2(\nu))}{\pi} \left[\frac{1 + (C/2)\alpha_{PMS}/\pi}{1 + C\alpha_{PMS}/\pi}\right], \tag{19}$$

where the PMS effective coupling $\alpha_{PMS}$ is a solution of the following transcendental equation

$$\frac{\pi}{\alpha_{PMS}} + C \ln\left(\frac{C\alpha_{PMS}/\pi}{1 + C\alpha_{PMS}/\pi}\right) + \frac{C/2}{1 + C\alpha_{PMS}/\pi} = \frac{\beta_0}{4} \ln\left(\frac{Q^2}{\Lambda^2}\right) - r(\nu) \tag{20}$$

with $C = \beta_1/(4\beta_0)$ and $\beta_1 = 102 - 38N_F/3$. At $\nu = 0$ one obtains in the $\overline{\rm MS}$-scheme $\omega_{PMS} = 0.23 - 0.20$ for $Q^2 = 1 - 100$ GeV$^2$, but, by the same reason as in the FAC case, the PMS effective coupling has a singularity at $\nu_0$. Thus, the application of the FAC and PMS scale setting approaches to the BFKL eigenvalue problem lead to difficulties

with the conformal weight dependence, an essential ingredient of BFKL calculations. The unphysical behavior of the FAC and PMS effective scales for jet production processes has been noted in Refs. [22].

Before making conclusions a few remarks are in order.

($i$) Since the BFKL equation can be interpreted as the "quantization" of a renormalization group equation [16], it follows that the effective scale should depend on the BFKL eigenvalue $\omega$, associated with the Lorentz spin, rather than on $\nu$. Thus, strictly speaking, one can use the above effective scales as function of $\nu$ only in "quasi-classical" approximation at large-$Q^2$. However, the present remarkable $Q^2$-stabilty leads us to expect that the results obtained with LO eigenfunctions may not change considerably for $t \neq 0$ due to the approximate conformal invariance. This issue will be discussed in more detail in the extended version of this work [23].

($ii$) There have been a number of recent papers which analyze the NLO BFKL predictions in terms of rapidity correlations [24], $t$-channel unitarity [25], angle-ordering [26], double transverse momentum logarithms [27] and BLM scale setting for deep inelastic structure functions [28]. A discussion of these topics within our approach will be deferred to Ref. [23].

To conclude, we have shown that the NLO corrections to the BFKL equation for the QCD Pomeron become controllable and meaningful provided one uses physical renormalization scales and schemes relevant to non-Abelian gauge theory. BLM optimal scale setting automatically sets the appropriate physical renormalization scale by absorbing the non-conformal $\beta$-dependent coefficients. The strong renormalization scheme dependence of the NLO corrections to BFKL resummation then largely disappears. This is in contrast to the unstable NLO results obtained in the conventional $\overline{\mathrm{MS}}$-scheme with arbitrary choice of renormalization scale. A striking feature of the NLO BFKL Pomeron intercept in the BLM approach is its very weak $Q^2$-dependence, which provides approximate conformal invariance. The new results presented here open new windows for applications of NLO BFKL resummation to high-energy phenomenology.


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